NATIONAL BUREAU OF STANDARDS REPORT

5320

SKELETON TABLES
FOR
MANUAL ON EXPERIMENTAL STATISTICS
FOR ORDNANCE ENGINEERS

A Report to

Office of Ordnance Research
Department of the Army

NBS

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

THE NATIONAL BUREAU OF STANDARDS

Functions and Activities

The functions of the National Bureau of Standards are set forth in the Act of Congress, March 3, 1901, as amended by Congress in Public Law 619, 1950. These include the development and maintenance of the national standards of measurement and the provision of means and methods for making measurements consistent with these standards; the determination of physical constants and properties of materials; the development of methods and instruments for testing materials, devices, and structures; advisory services to Government Agencies on scientific and technical problems; invention and development of devices to serve special needs of the Government; and the development of standard practices, codes, and specifications. The work includes basic and applied research, development, engineering, instrumentation, testing, evaluation, calibration services, and various consultation and information services. A major portion of the Bureau's work is performed for other Government Agencies, particularly the Department of Defense and the Atomic Energy Commission. The scope of activities is suggested by the listing of divisions and sections on the inside of the back cover.

Reports and Publications

The results of the Bureau's work take the form of either actual equipment and devices or published papers and reports. Reports are issued to the sponsoring agency of a particular project or program. Published papers appear either in the Bureau's own series of publications or in the journals of professional and scientific societies. The Bureau itself publishes three monthly periodicals, available from the Government Printing Office: The Journal of Research, which presents complete papers reporting technical investigations; the Technical News Bulletin, which presents summary and preliminary reports on work in progress; and Basic Radio Propagation Predictions, which provides data for determining the best frequencies to use for radio communications throughout the world. There are also five series of nonperiodical publications: The Applied Mathematics Series, Circulars, Handbooks, Building Materials and Structures Reports, and Miscellaneous Publications.

Information on the Bureau's publications can be found in NBS Circular 460, Publications of the National Bureau of Standards ($1.25) and its Supplement ($0.75), available from the Superintendent of Documents, Government Printing Office, Washington 25, D. C.

Inquiries regarding the Bureau's reports should be addressed to the Office of Technical Information, National Bureau of Standards, Washington 25, D. C.

NATIONAL BUREAU OF STANDARDS REPORT

NBS PROJECT — 1103-40-5146

NBS REPORT — 5320

5 August 1957

Skeleton Tables
for

Manual on Experimental Statistics
for Ordnance Engineers

Prepared by

Statistical Engineering Laboratory

A Report to

Office of Ordnance Research
Department of the Army

NBS

U. S. DEPARTMENT OF COMMERCE

NATIONAL BUREAU OF STANDARDS

N O T I C E

This report presents in skeleton form the tables which will eventually appear in the Manual on Experimental Statistics for Ordnance Engineers, as an aid to evaluation of drafts of various portions of the Manual being circulated for comment.

LIST OF TABLES

		Page
Ia	Cumulative Normal Distribution..................	1
Ib	Cumulative Normal Distribution..................	2
II	Percentiles of the t Distribution..............	3
III	Percentiles of the F Distribution..............	4
IV	Percentiles of q (Studentized Range)...........	5
V	Percentiles of the χ^2 Distribution...........	6
VI	Confidence Belts for Proportions...............	7
VII	Confidence Belts for the Correlation Coefficient..................................	8
VIII	Weighting Coefficients for Probit Analysis.....	9
IX	Maximum and Minimum Working Probits and Range..	10
X	Tolerance Factors for Normal Distributions.....	11
XI	Criteria for Rejection of Outlying Observations	12
XII	Percentiles of T(n) for the "Wilcoxon Signed-ranks Test".....................................	13
XIII	Probabilities Associated with the Mann-Whitney Test...	14
XIV	Percentiles of $U(n_1,n_2)$ for the "Mann-Whitney" Test...	15
XV	Tables for Distribution-free Tolerance Limits (Two-sided)....................................	16
XVI	Tables for Distribution-free Tolerance Limits (One-sided)....................................	17
XVII	Confidence Associated with a Tolerance Limit Statement......................................	18

List of Tables (Continued) Page

XVIIIa Table of Required Sample Sizes.................. 19 - 20

XVIIIb Table of Required Sample Sizes.................. 21 - 22

XIX Percentiles for $\varphi = (\bar{X}-m_0)/w$.................. 23

XX Percentiles for $\varphi' = \frac{\bar{X}_A-\bar{X}_B}{(w_A+w_B)/2}$ 24

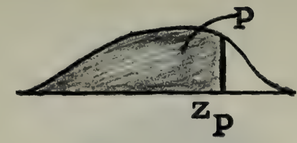
XXI Critical values of L for the Link-Wallace Test. 25 - 26

XXII Percentiles of $F' = w_A/w_B$..................... 27

XXIII Tables for Computing Confidence Limits for σ... 28

Table Ia

Cumulative Normal Distribution

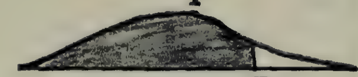


Values of P corresponding to z_p for the normal curve
z is the standard normal variable.

z_p	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
-3	.0013	.0010	.0007	.0005	.0003	.0002	.0002	.0001	.0001	.0000
-2	.0228	.0179	.0139	.0107	.0082	.0062	.0047	.0035	.0026	.0019
-1	.1587	.1357	.1151	.0968	.0808	.0668	.0548	.0446	.0359	.0287
-0	.5000	.4602	.4207	.3821	.3446	.3085	.2743	.2420	.2119	.1841
+0	.5000	.5398	.5793	.6179	.6554	.6915	.7257	.7580	.7881	.8159
1	.8413	.8643	.8849	.9032	.9192	.9332	.9452	.9554	.9641	.9713
2	.9772	.9821	.9861	.9893	.9918	.9938	.9953	.9965	.9974	.9981
3	.9987	.9990	.9993	.9995	.9997	.9998	.9998	.9999	.9999	1.0000

Table Ib

Cumulative Normal Distribution



Values of z_P Corresponding to P for the normal curve.
z is the standard normal variable

P	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.00	-	-2.33	-2.05	-1.88	-1.75	-1.64	-1.55	-1.48	-1.41	-1.34
.10	-1.28	-1.23	-1.18	-1.13	-1.08	-1.04	-0.99	-0.95	-0.92	-0.88
.20	-0.84	-0.81	-0.77	-0.74	-0.71	-0.67	-0.64	-0.61	-0.58	-0.55
.30	-0.52	-0.50	-0.47	-0.44	-0.41	-0.39	-0.36	-0.33	-0.31	-0.28
.40	-0.25	-0.23	-0.20	-0.18	-0.15	-0.13	-0.10	-0.08	-0.05	-0.03
.50	0.00	0.03	0.05	0.08	0.10	0.13	0.15	0.18	0.20	0.23
.60	0.25	0.28	0.31	0.33	0.36	0.39	0.41	0.44	0.47	0.50
.70	0.52	0.55	0.58	0.61	0.64	0.67	0.71	0.74	0.77	0.81
.80	0.84	0.88	0.92	0.95	0.99	1.04	1.08	1.13	1.18	1.23
.90	1.28	1.34	1.41	1.48	1.55	1.64	1.75	1.88	2.05	2.33

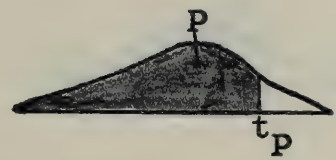
Special values

P	.001	.005	.010	.025	.050	.100
z_P	-3.090	-2.576	-2.326	-1.960	-1.645	-1.282

P	.999	.995	.990	.975	.950	.900
z_P	3.090	2.576	2.326	1.960	1.645	1.282

TABLE II

Percentiles of the t Distribution

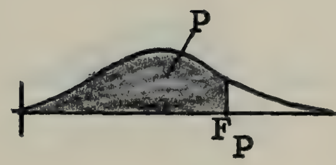


d.f.	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$
1	3.078	6.314	12.706	31.821	63.657
2					
⋮					
9			2.262	2.821	
⋮					
19		1.729	2.093		
⋮					
120					
∞	1.282	1.645	1.960	2.326	2.576

Use d.f. 1(1)30, 40, 60, 120, ∞. Values taken from Table A-5 Dixon and Massey "Introduction to Statistical Analysis," Second Edition, McGraw-Hill (1957).

TABLE III

Percentiles of the F Distribution



$$F_{.95}(n_1, n_2)$$

n_1 = degrees of freedom for numerator

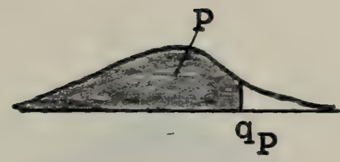
n_2 = degrees of freedom for denominator.

Reproduce from Dixon and Massey, Table A-7a, Second Edition, McGraw-Hill (1957).

Reproduce also $F_{.99}(n_1, n_2)$. This is Table A-7b of the above reference.

TABLE IV

Percentiles of q (Studentized Range)



$q = w/s$. w is the range of t observations, and ν is the number of degrees of freedom associated with the standard deviation s.

$q_{.95}$

ν \ t	2(1)20
1 (1) 20	

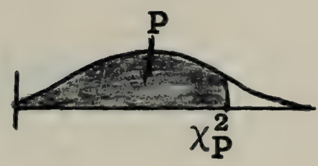
$q_{.99}$

ν \ t	2(1)20
1 (1) 20	

Values for above tables taken from Table A-18, Dixon and Massey, Second Edition (1957).

TABLE V

Percentiles of the χ^2 Distribution



Values of χ_P^2 corresponding to P

d.f.	$\chi^2_{.90}$	$\chi^2_{.95}$	$\chi^2_{.975}$	$\chi^2_{.99}$	$\chi^2_{.995}$

For large degrees of freedom,

$$\chi_P^2 \doteq (z_P + \sqrt{2(\text{d.f.})-1})^2/2$$

where z_P is given in Table I.

---

d.f. = 1(1) 16, 18, 20, 24, 30, 40, 60, 120

Values taken from Table A-6a, Dixon and Massey, McGraw-Hill Second Edition (1957).

TABLE VI

Confidence Belts for Proportions

(Change labels, Ordinate label - P
Abscissae label - p)

1st chart	Confidence coefficient .90
2nd chart	Confidence coefficient .95
3rd chart	Confidence coefficient .99

Charts 1,2,3 are reproduced from Dixon and Massey, p. 414, 415, 416, Second Edition, McGraw-Hill (1957).

TABLE VII

Confidence Belts for the Correlation Coefficient
(confidence coefficient .95)

Reproduced from Table A-27, Dixon and Massey, Second Edition, McGraw-Hill (1957).

TABLE VIII

Weighting Coefficients for Probit Analysis

Y	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	0.001	0.001	0.001	0.002	0.002	0.003	0.005	0.006	0.008	0.011
2	0.015	0.019	0.025	0.031	0.040	0.050	0.062	0.076	0.092	0.110
3	0.131	0.154	0.180	0.208	0.238	0.269	0.302	0.336	0.370	0.405
4	0.439	0.471	0.503	0.532	0.558	0.581	0.601	0.616	0.627	0.634
5	0.637	0.634	0.627	0.616	0.601	0.581	0.558	0.532	0.503	0.471
6	0.439	0.405	0.370	0.336	0.302	0.269	0.238	0.208	0.180	0.154
7	0.131	0.110	0.092	0.076	0.062	0.050	0.040	0.031	0.025	0.019
8	0.015	0.011	0.008	0.006	0.005	0.003	0.002	0.002	0.001	0.001

Values obtained from page 32, Finney, Cambridge University Press (1952).

TABLE IX

Maximum and Minimum Working Probits and Range

Expected probit Y	Minimum working probit y_0	Range 1/z	Maximum working probit y_{100}	Expected probit Y
1.1	0.8579	5034	9.1421	8.9
1.2	0.9522	3425	9.0478	8.8
1.3	1.0462	2354	8.9538	8.7
1.4	1.1400	1634	8.8600	8.6
1.5	1.2334	1146	8.7666	8.5
1.6	1.3266	811.5	8.6734	8.4
1.7	1.4194	580.5	8.5806	8.3
1.8	1.5118	419.4	8.4882	8.2
1.9	1.6038	306.1	8.3962	8.1
2.0	1.6954	225.6	8.3046	8.0
2.1	1.7866	168.00	8.2134	7.9
2.2	1.8772	126.34	8.1228	7.8
2.3	1.9673	95.96	8.0327	7.7
2.4	2.0568	73.62	7.9432	7.6
2.5	2.1457	57.05	7.8543	7.5
2.6	2.2339	44.654	7.7661	7.4
2.7	2.3214	35.302	7.6786	7.3
2.8	2.4081	28.189	7.5919	7.2
2.9	2.4938	22.736	7.5062	7.1
3.0	2.5786	18.5216	7.4214	7.0
3.1	2.6624	15.2402	7.3376	6.9
3.2	2.7449	12.6662	7.2551	6.8
3.3	2.8261	10.6327	7.1739	6.7
3.4	2.9060	9.0154	7.0940	6.6
3.5	2.9842	7.7210	7.0158	6.5
3.6	3.0606	6.6788	6.9394	6.4
3.7	3.1351	5.8354	6.8649	6.3
3.8	3.2074	5.1497	6.7926	6.2
3.9	3.2773	4.5903	6.7227	6.1
4.0	3.3443	4.1327	6.6557	6.0
4.1	3.4083	3.7582	6.5917	5.9
4.2	3.4687	3.4519	6.5313	5.8
4.3	3.5251	3.2025	6.4749	5.7
4.4	3.5770	3.0010	6.4230	5.6
4.5	3.6236	2.8404	6.3764	5.5
4.6	3.6643	2.7154	6.3357	5.4
4.7	3.6982	2.6220	6.3018	5.3
4.8	3.7241	2.5573	6.2759	5.2
4.9	3.7407	2.5192	6.2593	5.1
5.0	3.7467	2.5066	6.2533	5.0

TABLE X

Tolerance Factors for Normal Distributions

Factors K such that the probability is γ that at least a proportion P of the distribution will be included between $\overline{X} + Ks$, where $\overline{X}$ and s are estimates of the mean and standard deviation computed from a sample of n.

Use format as pp. 102-107, "Techniques of Statistical Analysis", Eisenhart, Hastay and Wallis, McGraw-Hill (1947). Abridge, using n = 2(1)20, 25, 30(10)100, 100(100)600, 800,1000,∞ .

TABLE XI

Criteria for Rejection of Outlying Observations

Statistic	Number of observations n	Critical values						
		α= .30	α= .20	α= .10	α= .05	α'= .02	α'= .01	α'= .005

Reproduced from Table A-8e, Dixon and Massey, McGraw-Hill, Second Edition (1957).

TABLE XII

Percentiles of $T(n)$ for the "Wilcoxon Signed-ranks Test"

n	$T_{.025}(n)$	$T_{.01}(n)$	$T_{.005}(n)$
6	0	-	-
7	2	0	-
8	4	2	0
9	6	3	2
10	8	5	3
11	11	7	5
12	14	10	7
13	17	13	10
14	21	16	13
15	25	20	16
16	30	24	20
17	35	28	23
18	40	33	28
19	46	38	32
20	52	43	38
21	59	49	43
22	66	56	49
23	73	62	55
24	81	69	61
25	89	77	68

Adapted from Table II, F. Wilcoxon, 1949, "Some rapid approximate statistical procedures", New York:American Cyanamid Company, p. 14.

(See also, Table G, p. 254, Siegel, "Non-parametric Statistics", McGraw-Hill (1956).

For large n,

$$T_p(n) = \frac{n(n+1)}{4} - z_{1-p} \sqrt{\frac{n(n+1)(2n+1)}{24}} \quad \text{approximately}$$

where z is given in Table I.

TABLE XIII

Probabilities Associated with the Mann-Whitney Test

Probabilities associated with values as small as U.

Reproduce tables of Mann and Whitney from Annals of Mathematical Statistics Volume 18, (1947), pp. 52-54. (Same tables are in Siegel, McGraw-Hill, 1956, Table J). Eliminate last two columns of last table. Put note above each table:

"n_1 is the smaller of n_A, n_B,

n_2 is the larger of n_A, n_B."

TABLE XIV

Percentiles of $U(n_1,n_2)$ for the "Mann Whitney" Test

a) $U_{.001}(n_1,n_2)$ Reproduce Table K_1 p. 274 of *.

b) $U_{.01}(n_1,n_2)$ Reproduce Table K_2 p. 275 of *.

c) $U_{.025}(n_1,n_2)$ Reproduce Table K_3 p. 276 of *.

d) $U_{.05}(n_1,n_2)$ Reproduce Table K_4 p. 277 of *.

Put note above each table: "n_2,n_1 are the larger and smaller respectively of n_A,n_B."

*) Siegel, McGraw-Hill, 1956.

NOTE: for $n > 20$,

$$U_{\alpha/2} = \frac{n_A n_B}{2} - z_{1-\alpha/2}\sqrt{\frac{n_A n_B(n_A+n_B+1)}{12}}$$

approximately where z is given in Table I.

TABLE XV

Tables for Distribution-free Tolerance Limits (Two-sided)

Values (r,s) such that we may assert with confidence at least γ that 100P percent of a population lies between the r^{th} smallest and the s^{th} largest of a random sample of n from that population (no assumption of normality required).

n \ P	γ = 0.75				γ = 0.90				γ = 0.95				γ = 0.99			
	.75	.90	.95	.99	.75	.90	.95	.99	.75	.90	.95	.99	.75	.90	.95	.99
50	5,5	2,1	–	–	5,4	1,1	–	–	4,4	1,1	–	–	3,3	–	–	–
55	6,6	2,2	1,1	–	5,5	2,1	–	–	5,4	1,1	–	–	4,3	–	–	–
60	7,6	2,2	1,1	–	6,5	2,1	–	–	5,5	1,1	–	–	4,4	–	–	–
65	7,7	3,2	1,1	–	6,6	2,2	–	–	6,5	2,1	–	–	5,4	1,1	–	–
70	8,7	3,2	1,1	–	7,6	2,2	–	–	6,6	2,1	–	–	5,5	1,1	–	–
75	8,8	3,3	1,1	–	7,7	2,2	–	–	7,6	2,1	–	–	5,5	1,1	–	–
80	9,8	3,3	2,2	–	8,7	3,2	1,1	–	7,7	2,2	–	–	6,5	1,1	–	–
85	10,9	4,3	2,2	–	8,8	3,2	1,1	–	8,7	2,2	–	–	6,6	2,1	–	–
90	10,10	4,3	2,2	–	9,8	3,2	1,1	–	8,8	3,2	–	–	7,6	2,1	–	–
95	11,10	4,3	2,2	–	9,9	3,3	1,1	–	9,8	3,2	1,1	–	7,7	2,1	–	–
100	11,11	4,4	2,2	–	10,10	3,3	1,1	–	9,9	3,2	1,1	–	8,7	2,2	–	–
110	12,12	5,4	2,2	–	11,11	4,3	2,1	–	10,10	3,3	1,1	–	9,8	2,2	–	–
120	14,13	5,5	2,2	–	12,12	4,4	2,1	–	11,11	4,3	1,1	–	10,9	3,2	–	–
130	15,14	6,5	3,2	–	13,13	5,4	2,1	–	13,12	4,4	2,1	–	11,10	3,3	1,1	–
140	16,15	6,6	3,2	–	14,14	5,5	2,2	–	14,13	4,4	2,1	–	12,11	3,3	1,1	–
150	17,17	6,6	3,3	–	16,15	5,5	2,2	–	15,14	5,4	2,1	–	13,13	4,3	1,1	–
170	20,19	7,7	4,3	–	18,17	6,6	3,2	–	17,16	6,5	2,2	–	15,15	5,4	2,1	–
200	23,23	9,8	4,4	–	21,21	8,7	3,3	–	20,20	7,6	3,2	–	18,18	6,5	2,2	–
300	35,35	13,13	6,6	1,1	33,32	12,11	5,5	–	32,31	11,11	5,4	–	29,29	10,9	4,3	–
400	47,47	18,18	9,8	2,1	45,44	16,16	8,7	1,1	43,43	15,15	7,6	–	40,40	14,13	6,5	–
500	59,59	23,22	11,11	2,1	57,56	21,20	10,9	1,1	55,54	20,19	9,8	1,1	52,51	18,17	7,7	–
600	72,71	28,27	13,13	2,2	68,68	26,25	12,11	2,1	67,66	24,24	11,10	1,1	63,63	22,22	9,9	–
700	84,83	33,32	16,15	3,2	80,80	30,30	14,14	2,2	78,78	29,28	13,13	2,1	75,74	26,26	11,11	1,1
800	96,96	37,37	18,18	3,3	92,92	35,34	16,16	3,2	90,90	33,33	15,15	2,2	86,86	31,30	13,13	1,1
900	108,108	42,42	21,20	4,3	104,104	40,39	19,18	3,2	102,102	38,37	18,17	2,2	98,97	35,35	15,15	2,1
1000	121,120	47,47	23,22	4,4	117,116	44,44	21,20	3,3	114,114	43,42	20,19	3,2	110,109	40,39	18,17	2,1

TABLE XVI

Tables for Distribution-free Tolerance Limits (One-sided)

Largest values of m such that we may assert with confidence at least γ that 100P percent of a population lies below the m^{th} largest (or above the m^{th} smallest) of a random sample of n from that population (no assumption of normality required).

	$\gamma = 0.75$				$\gamma = 0.90$				$\gamma = 0.95$				$\gamma = 0.99$			
n \ P	.75	.90	.95	.99	.75	.90	.95	.99	.75	.90	.95	.99	.75	.90	.95	.99
50	10	3	1	-	9	2	1	-	8	2	-	-	6	1	-	-
55	12	4	2	-	10	3	1	-	9	2	-	-	7	1	-	-
60	13	4	2	-	11	3	1	-	10	2	1	-	8	1	-	-
65	14	5	2	-	12	4	1	-	11	3	1	-	9	2	-	-
70	15	5	2	-	13	4	1	-	12	3	1	-	10	2	-	-
75	16	6	2	-	14	4	1	-	13	3	1	-	10	2	-	-
80	17	6	3	-	15	5	2	-	14	4	1	-	11	2	-	-
85	19	7	3	-	16	5	2	-	15	4	1	-	12	3	-	-
90	20	7	3	-	17	5	2	-	16	5	1	-	13	3	1	-
95	21	7	3	-	18	6	2	-	17	5	2	-	14	3	1	-
100	22	8	3	-	20	6	2	-	18	5	2	-	15	4	1	-
110	24	9	4	-	22	7	3	-	20	6	2	-	17	4	1	-
120	27	10	4	-	24	8	3	-	22	7	2	-	19	5	1	-
130	29	11	5	-	26	9	3	-	25	8	3	-	21	6	2	-
140	31	12	5	1	28	10	4	-	27	8	3	-	23	6	2	-
150	34	12	6	1	31	10	4	-	29	9	3	-	26	7	2	-
170	39	14	7	1	35	12	5	-	33	11	4	-	30	9	3	-
200	46	17	8	1	42	15	6	-	40	13	5	-	36	11	4	-
300	70	26	12	2	65	23	10	1	63	22	9	1	58	19	7	-
400	94	36	17	3	89	32	15	2	86	30	13	1	80	27	11	-
500	118	45	22	3	113	41	19	2	109	39	17	2	103	35	14	1
600	143	55	26	4	136	51	23	3	133	48	21	2	126	44	18	1
700	167	65	31	5	160	60	28	4	156	57	26	3	149	52	22	2
800	192	74	36	6	184	69	32	5	180	66	30	4	172	61	26	2
900	216	84	41	7	208	79	37	5	204	75	35	4	195	70	30	3
1000	241	94	45	8	233	88	41	6	228	85	39	5	219	79	35	3

TABLE XVII

Confidence Associated with a Tolerance Limit Statement

Confidence γ with which we may assert that 100P percent of the population lies between the largest and smallest of a random sample of n from that population (continuous distribution assumed).

n	P=.75	P=.90	P=.95	P=.99	n	P=.75	P=.90	P=.95	P=.99
3	.16	.03	.01	.00	17	.95	.52	.21	.01
4	.26	.05	.01	.00	18	.96	.55	.23	.01
5	.37	.08	.02	.00	19	.97	.58	.25	.02
6	.47	.11	.03	.00	20	.98	.61	.26	.02
7	.56	.15	.04	.00	25	.99	.73	.36	.03
8	.63	.19	.06	.00	30	1.00-	.82	.45	.04
9	.70	.23	.07	.00	40		.92	.60	.06
10	.76	.26	.09	.00	50		.97	.72	.09
11	.80	.30	.10	.01	60		.99	.81	.12
12	.84	.34	.12	.01	70		.99	.87	.16
13	.87	.38	.14	.01	80		1.00-	.91	.19
14	.90	.42	.15	.01	90			.94	.23
15	.92	.45	.17	.01	100			.96	.26
16	.94	.49	.19	.01					

TABLE XVIIIa

Table of Required Sample Sizes

Sample size required for detecting, with probability $1-\beta$, whether the average m of a new product differs from the standard m_0 (or whether two product averages m_A and m_B differ).

$$d = \frac{m-m_0}{\sigma} \quad \left(\text{or } d = \frac{m_A - m_B}{\sqrt{\sigma_A^2+\sigma_B^2}}\right.$$ if we are comparing two products).

The standard deviations are assumed to be known.

$\alpha = .01$

d \ $1-\beta$	.50	.60	.70	.80	.90	.95	.99
.1	664	801	962	1168	1488	1782	2404
.2	166	201	241	292	372	446	601
.4	42	51	61	73	93	112	151
.6	19	23	27	33	42	50	67
.8	11	13	16	19	24	28	38
1.0	7	9	10	12	15	18	25
1.2	5	6	7	9	11	13	17
1.4	4	5	5	6	8	10	13
1.6	3	4	4	5	6	7	10
1.8	3	3	3	4	5	6	8
2.0	2	3	3	3	4	5	7
3.0	1	1	2	2	2	2	3

If we must estimate σ from our sample, and use Student's t then we should add 4 to the tabulated values to obtain the approximate required sample size. (If we are comparing two product averages, add 2 to the tabulated values, For this case, we must have $\sigma_A = \sigma_B$).

TABLE XVIIIa (Continued)

$\alpha = .05$

d \ 1-β	.50	.60	.70	.80	.90	.95	.99
.1	385	490	618	785	1051	1300	1838
.2	97	123	155	197	263	325	460
.4	25	31	39	50	66	82	115
.6	11	14	18	22	30	37	52
.8	7	8	10	13	17	21	29
1.0	4	5	7	8	11	13	19
1.2	3	4	5	6	8	10	13
1.4	2	3	4	5	6	7	10
1.6	2	2	3	4	5	6	8
1.8	2	2	2	3	4	5	6
2.0	1	2	2	2	3	4	5
3.0	1	1	1	1	2	2	3

If we must estimate σ from our sample and use Student's t, then we should add 2 to the tabulated values to obtain the approximate required sample size. (If we are comparing two produce averages, add 1 to the tabulated values).

TABLE XVIIIb

Table of Required Sample Sizes

Sample size required for detecting with probability $1-\alpha$ whether

a) the average m of a new product exceeds that of a standard m_0

b) the average m of a new product is less than that of a standard m_0

c) the average of a specified product m_A exceeds the average of another specified product m_B.

The standard deviations are assumed to be known

a) $d = \frac{m-m_0}{\sigma}$

b) $d = \frac{m_0-m}{\sigma}$

c) $d = \frac{m_A-m_B}{\sqrt{\sigma_A^2+\sigma_B^2}}$

$\alpha = .01$

d \ $1-\beta$	.50	.60	.70	.80	.90	.95	.99
.1	542	666	813	1004	1302	1578	2165
.2	136	167	204	251	326	395	542
.4	34	42	51	63	82	99	136
.6	16	19	23	28	37	44	61
.8	9	11	13	16	21	25	34
1.0	6	7	9	11	14	16	22
1.2	4	5	6	7	10	11	16
1.4	3	4	5	6	7	9	12
1.6	3	3	4	4	6	7	9
1.8	2	3	3	4	5	5	7
2.0	2	2	3	3	4	4	6
3.0	1	1	1	2	2	2	3

If we must estimate σ from our sample, and use Student's t, then we should add 3 to the tabulated values to obtain the approximate required sample size. (If we are comparing two product averages, add 2 to the tabulated values. For this case, we must have $\sigma_A = \sigma_B$).

Table XVIIIb (Continued)

$\alpha = .05$

d \ $1-\beta$	.50	.60	.70	.80	.90	.95	.99
.1	271	361	471	619	857	1083	1578
.2	68	91	118	155	215	271	395
.4	17	23	30	39	54	68	99
.6	8	11	14	18	24	31	44
.8	5	6	8	10	14	17	25
1.0	3	4	5	7	9	11	16
1.2	2	3	4	5	6	8	11
1.4	2	2	3	4	5	6	9
1.6	2	2	2	3	4	5	7
1.8	1	2	2	2	3	4	5
2.0	1	1	2	2	3	3	4
3.0	1	1	1	1	1	2	2

If we must estimate σ from our sample, and use Student's t, then we should add 2 to the tabulated values to obtain the approximate required sample size. (If we are comparing two product averages, add 1 to the tabulated values. For this case, we must have $\sigma_A = \sigma_B$).

TABLE XIX

Percentiles for $\varphi = \dfrac{\overline{X}-m_0}{w}$

n	$\varphi_{.95}$	$\varphi_{.975}$	$\varphi_{.99}$	$\varphi_{.995}$	$\varphi_{.999}$	$\varphi_{.9995}$
2 (1) 20						

Reproduced from Table A-8c(1), Dixon and Massey, Second Edition, McGraw-Hill (1957).

TABLE XX

Percentiles for $\varphi' = \dfrac{\bar{X}_A - \bar{X}_B}{1/2(w_A + w_B)}$

$n = n_A = n_B$	$\varphi'_{.95}$	$\varphi'_{.975}$	$\varphi'_{.99}$	$\varphi'_{.995}$	$\varphi'_{.999}$	$\varphi'_{.9995}$
2						
3						
⋮						
20						

Reproduced from Table A-8c(2), Dixon and Massey, Second Edition, McGraw-Hill (1957).

TABLE XXI

Critical values of L for the Link-Wallace Test

$\alpha = .05$

t = number of groups = number of ranges

n = number in group = number per range

	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	3.43	2.37	1.78	1.40	1.16	1.00	.87	.78	.70	.66	.63	.58	.50	.47	.44	.42	.40	.38	.36
3	1.91	1.44	1.13	.94	.80	.70	.62	.56	.51	.47	.43	.40	.38	.36	.33	.32	.30	.29	.27
4	1.63	1.25	1.01	.84	.72	.63	.57	.51	.47	.43	.40	.37	.35	.33	.31	.29	.28	.27	.25
5	1.53	1.19	.96	.81	.70	.61	.55	.50	.45	.42	.39	.36	.34	.32	.30	.29	.27	.26	.25
6	1.50	1.18	.95	.80	.69	.61	.55	.49	.45	.42	.39	.36	.34	.32	.30	.29	.27	.26	.25
7	1.49	1.17	.95	.80	.69	.61	.55	.50	.45	.42	.39	.36	.34	.32	.30	.29	.28	.26	.25
8	1.49	1.17	.96	.81	.70	.62	.55	.50	.46	.42	.39	.37	.35	.33	.31	.29	.28	.27	.25
9	1.50	1.18	.97	.82	.71	.62	.56	.51	.47	.43	.40	.37	.35	.33	.31	.30	.28	.27	.26
10	1.52	1.20	.98	.83	.72	.63	.57	.52	.47	.44	.41	.38	.35	.34	.32	.30	.29	.27	.26
11	1.54	1.21	.99	.84	.73	.64	.58	.52	.48	.44	.41	.38	.36	.34	.32	.31	.29	.28	.27
12	1.56	1.23	1.00	.85	.74	.65	.59	.53	.49	.45	.42	.39	.37	.35	.33	.31	.30	.28	.27
13	1.58	1.25	1.02	.86	.75	.66	.59	.54	.49	.46	.42	.40	.37	.35	.33	.32	.30	.29	.27
14	1.60	1.26	1.03	.87	.76	.67	.60	.55	.50	.46	.43	.40	.38	.36	.34	.32	.31	.29	.28
15	1.62	1.28	1.05	.89	.77	.68	.61	.56	.51	.47	.44	.41	.38	.36	.34	.33	.31	.30	.28
16	1.64	1.30	1.06	.90	.78	.69	.62	.56	.52	.48	.44	.41	.39	.37	.35	.33	.31	.30	.29
17	1.66	1.31	1.08	.91	.79	.70	.63	.57	.52	.48	.45	.42	.39	.37	.35	.33	.32	.30	.29
18	1.68	1.33	1.09	.92	.80	.71	.64	.58	.53	.49	.46	.43	.40	.38	.36	.34	.32	.31	.30
19	1.70	1.34	1.10	.93	.81	.72	.65	.59	.54	.50	.46	.43	.40	.38	.36	.34	.33	.31	.30
20	1.72	1.36	1.11	.95	.82	.73	.65	.59	.54	.50	.47	.44	.41	.39	.37	.35	.33	.32	.30

Table XXI (Continued)

$\alpha = .01$

t = number of groups = number of ranges

n = number in group = number per range

	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	7.92	4.42	2.96	2.06	1.69	1.39	1.20	1.03	.91	.82	.75	.68	.63	.59	.55	.51	.48	.46	.43
3	3.14	2.14	1.57	1.25	1.04	.89	.78	.69	.62	.57	.52	.48	.45	.42	.39	.37	.35	.34	.32
4	2.47	1.74	1.33	1.08	.91	.78	.69	.62	.56	.51	.47	.44	.41	.38	.36	.34	.32	.31	.29
5	2.24	1.60	1.24	1.02	.86	.75	.66	.59	.54	.49	.46	.42	.40	.37	.35	.33	.31	.30	.29
6	2.14	1.55	1.21	.99	.85	.74	.65	.59	.53	.49	.45	.42	.39	.37	.35	.33	.31	.30	.28
7	2.10	1.53	1.21	.99	.84	.74	.65	.59	.53	.49	.45	.42	.40	.37	.35	.33	.32	.30	.29
8	2.08	1.52	1.21	.99	.85	.74	.66	.59	.54	.50	.46	.43	.40	.37	.35	.33	.32	.30	.29
9	2.09	1.53	1.22	1.00	.85	.75	.66	.60	.54	.50	.46	.43	.40	.38	.36	.34	.32	.31	.29
10	2.10	1.55	1.23	1.01	.86	.75	.67	.61	.55	.51	.47	.44	.41	.38	.36	.34	.33	.31	.30
11	2.11	1.56	1.24	1.02	.88	.77	.68	.61	.56	.51	.48	.44	.42	.39	.37	.35	.33	.32	.30
12	2.13	1.58	1.25	1.03	.89	.78	.69	.62	.57	.52	.48	.45	.42	.40	.37	.35	.34	.32	.31
13	2.15	1.60	1.27	1.05	.90	.79	.70	.63	.58	.53	.49	.46	.43	.40	.38	.36	.34	.33	.31
14	2.18	1.62	1.28	1.06	.91	.80	.71	.64	.58	.54	.50	.46	.43	.41	.39	.37	.35	.33	.32
15	2.20	1.64	1.30	1.08	.92	.81	.72	.65	.59	.54	.50	.47	.44	.41	.39	.37	.35	.34	.32
16	2.22	1.65	1.31	1.09	.93	.82	.73	.66	.60	.55	.51	.48	.45	.42	.40	.38	.36	.34	.32
17	2.24	1.67	1.33	1.11	.95	.83	.74	.67	.61	.56	.52	.48	.45	.43	.40	.38	.36	.34	.33
18	2.27	1.69	1.34	1.12	.96	.84	.75	.68	.62	.57	.53	.49	.46	.43	.41	.39	.37	.35	.33
19	2.30	1.71	1.36	1.14	.97	.85	.76	.68	.62	.57	.53	.50	.46	.44	.41	.39	.37	.35	.34
20	2.32	1.73	1.38	1.15	.98	.86	.77	.69	.63	.58	.54	.50	.47	.44	.42	.40	.38	.36	.34

TABLE XXII

Percentiles of $F' = \frac{w_A}{w_B}$

n_B		n_A								
		2	3	4	5	6	7	8	9	10
2	.005 .01 .025 .05	.0078 .0157 .039 .079								
3	.005 .01 .025 .05									
4	"									
5	"									
6	"									
7	"									
8	"									
9	"									
10	"									

Taken from Table A-8d, Dixon and Massey, Second Edition, (1957).

TABLE XXIII

Tables for Computing Confidence Limits for σ

Degrees of Freedom ν	$A_{.05}$	$A_{.95}$	$A_{.025}$	$A_{.975}$	$A_{.01}$	$A_{.99}$	$A_{.005}$	$A_{.995}$
1	.5103	15.947	.4461	31.910	.3882	79.786	.3562	159.576
2	.5778	4.415	.5207	6.285	.4660	9.975	.4344	14.124
3	.6196	2.920	.5665	3.729	.5142	5.111	.4834	6.467
4	.6493	2.372	.5992	2.874	.5489	3.669	.5188	4.396
5	.6721	2.089	.6242	2.453	.5757	3.003	.5464	3.485
6	.6903	1.915	.6444	2.202	.5974	2.623	.5688	2.980
7	.7054	1.797	.6612	2.035	.6155	2.377	.5875	2.660
8	.7183	1.711	.6754	1.916	.6310	2.204	.6037	2.439
9	.7293	1.645	.6878	1.826	.6445	2.076	.6177	2.278
10	.7391	1.593	.6987	1.755	.6564	1.977	.6301	2.154
11	.7477	1.551	.7084	1.698	.6670	1.898	.6412	2.056
12	.7554	1.515	.7171	1.651	.6765	1.833	.6512	1.976
13	.7624	1.485	.7250	1.611	.6852	1.779	.6603	1.909
14	.7688	1.460	.7321	1.577	.6931	1.733	.6686	1.854
15	.7747	1.437	.7387	1.548	.7004	1.694	.6762	1.806
20	.7979	1.358	.7650	1.444	.7297	1.556	.7071	1.640
25	.8149	1.308	.7843	1.380	.7511	1.473	.7299	1.542
30	.8279	1.274	.7991	1.337	.7678	1.416	.7477	1.475
40	.8470	1.228	.8210	1.279	.7925	1.343	.7740	1.390
50	.8606	1.199	.8367	1.243	.8103	1.297	.7931	1.337
60	.8710	1.179	.8487	1.217	.8239	1.265	.8078	1.299
70	.8793	1.163	.8583	1.198	.8349	1.241	.8196	1.272
80	.8861	1.151	.8662	1.183	.8439	1.222	.8293	1.250
90	.8919	1.141	.8728	1.171	.8515	1.207	.8376	1.233
100	.8968	1.133	.8785	1.161	.8581	1.195	.8446	1.219

For large degrees of freedom, the following approximate formula may be used

$$A_p = \sqrt{2\nu/(z_p + \sqrt{2\nu-1})^2}$$

U. S. DEPARTMENT OF COMMERCE
Sinclair Weeks, *Secretary*

NATIONAL BUREAU OF STANDARDS
A. V. Astin, *Director*

THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards at its headquarters in Washington, D. C., and its major field laboratories in Boulder, Colorado, is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section carries out specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant reports and publications, appears on the inside front cover of this report.

WASHINGTON, D. C.

Electricity and Electronics. Resistance and Reactance. Electron Tubes. Electrical Instruments. Magnetic Measurements. Dielectrics. Engineering Electronics. Electronic Instrumentation. Electrochemistry.

Optics and Metrology. Photometry and Colorimetry. Optical Instruments. Photographic Technology. Length. Engineering Metrology.

Heat and Power. Temperature Physics. Thermodynamics. Cryogenic Physics. Rheology and Lubrication. Engine Fuels.

Atomic and Radiation Physics. Spectroscopy. Radiometry. Mass Spectrometry. Solid State Physics. Electron Physics. Atomic Physics. Nuclear Physics. Radioactivity. X-rays. Betatron. Nucleonic Instrumentation. Radiological Equipment. AEC Radiation Instruments.

Chemistry. Organic Coatings. Surface Chemistry. Organic Chemistry. Analytical Chemistry. Inorganic Chemistry. Electrodeposition. Gas Chemistry. Physical Chemistry. Thermochemistry. Spectrochemistry. Pure Substances.

Mechanics. Sound. Mechanical Instruments. Fluid Mechanics. Engineering Mechanics. Mass and Scale. Capacity, Density, and Fluid Meters. Combustion Controls.

Organic and Fibrous Materials. Rubber. Textiles. Paper. Leather. Testing and Specifications. Polymer Structure. Organic Plastics. Dental Research.

Metallurgy. Thermal Metallurgy. Chemical Metallurgy. Mechanical Metallurgy. Corrosion. Metal Physics.

Mineral Products. Engineering Ceramics. Glass. Refractories. Enameled Metals. Concreting Materials. Constitution and Microstructure.

Building Technology. Structural Engineering. Fire Protection. Heating and Air Conditioning. Floor, Roof, and Wall Coverings. Codes and Specifications.

Applied Mathematics. Numerical Analysis. Computation. Statistical Engineering. Mathematical Physics.

Data Processing Systems. SEAC Engineering Group. Components and Techniques. Digital Circuitry. Digital Systems. Analogue Systems. Application Engineering.

- Office of Basic Instrumentation
- Office of Weights and Measures

BOULDER, COLORADO

Cryogenic Engineering. Cryogenic Equipment. Cryogenic Processes. Properties of Materials. Gas Liquefaction.

Radio Propagation Physics. Upper Atmosphere Research. Ionospheric Research. Regular Propagation Services. Sun-Earth Relationships.

Radio Propagation Engineering. Data Reduction Instrumentation. Modulation Systems. Navigation Systems. Radio Noise. Tropospheric Measurements. Tropospheric Analysis. Radio Systems Application Engineering.

Radio Standards. Radio Frequencies. Microwave Frequencies. High Frequency Electrical Standards. Radio Broadcast Service. High Frequency Impedance Standards. Calibration Center. Microwave Physics. Microwave Circuit Standards.

NBS